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HUBO Formulation For Task Allocation Optimization in Hybrid Optical-Electrical Networks

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***Abstract*—In supercomputing, large problems are divided into multiple computational tasks, which are then executed across the supercomputer's network. The process of mapping these computational tasks onto the network constitutes an embedding, which is essentially equivalent to a combinatorial optimization problem of graph embedding. For such graph embedding, attaching an optical switch to host graph ensures that, even if the embedding of one edge among those connected to each node of guest graph fails, a successful embedding can still be achieved for optical switch. Therefore, introducing optical switch is expected to provide new opportunities for distributed allocation of computational tasks over the supercomputer's network. Although solving large-scale combinatorial optimization problems on class computers remains highly challenging, a variety of approaches have been actively explored in recent years. Among these approaches, Quadratic Unconstrained Binary Optimization (QUBO) is especially suitable for being solved by quantum computers based on quantum annealing. When dealing with higher-order polynomial terms, the QUBO formulation necessitates the introduction of a large number of auxiliary variables, thereby incurring a significant computational cost. This study addresses graph embedding using optical switch by High-order Unconstrained Binary Optimization (HUBO), which allows for the direct treatment of higher-order polynomial terms. In this study, we formulate the graph embedding problem without optical switch as a QUBO, where the objective function is expressed as the sum of linear, quadratic terms and formulate the graph embedding problem with optical switch as a HUBO, where the objective function is expressed as the sum of linear, quadratic, cubic, and quartic terms. In our experiments, we embedded the hypercube graph and the torus graph into random regular graphs with optical switch and without optical switch. It was observed that the QUBO and HUBO models we proposed can solve graph embedding problem and attaching optical switch to host graph appears to be more effective.**

***Index Terms*—supercomputer's network, graph embedding, optical switch, QUBO, HUBO**

I. Introduction

A. QUBO and HUBO

QUBO is a formulation of an optimization problem in which the objective function is quadratic, no explicit constrains are specified, necessary constraints are incorporated in to the objective as penalty terms. All variables are restricted to binary values of 0 or 1. Many optimization problems can be formulated into QUBO models [1], [2]. The general form of a QUBO model is given by

$$E = \sum_{i=1}^{n} Q_{ii} x_i + \sum_{i<j} Q_{ij} x_i x_j,$$

where $x_i \in \{0, 1\}$ are binary variables, Q_{ii}represents the linear coefficients. $Q_{ij}(i < j)$ represents the quadratic interaction coefficients. Consider a QUBO with four binary variables $x_1, x_2, x_3, x_4 \in \{0, 1\}$. The objective function is given as follow:

$$E = -5x_1 - 3x_2 - 8x_3 - 6x_4 + 4x_1x_2 + 8x_1x_3 + 5.$$

The QUBO problem is to find $x_1, x_2, x_3, x_4 \in \{0, 1\}$ that minimize $E(x_1, x_2, x_3, x_4)$. By exhaustively evaluating all 2^4 possible assignments, we find that the minimum value of E is -12, which is attained uniquely when $(x_1, x_2, x_3, x_4) = (0, 1, 1, 1)$. E is QUBO formulations are unable to represent polynomial terms of order three or higher. Quadratization of such terms necessitates the introduction of auxiliary variables. For large-scale problems, the introduction of a substantial number of auxiliary variables becomes necessary, thereby incurring considerable computational overhead. However, HUBO formulations can directly accommodate higher-order polynomial terms, thus avoiding the introduction of auxiliary variables required for quadratization. Many studies have focused on transforming HUBO problems into QUBO problems by introducing additional auxiliary variables [10]. However, in this work, we use HUBO directly. A HUBO problem is generally formulated as

$$E = \sum_{i=1}^{n} C_i x_i + \sum_{i<j} C_{ij} x_i x_j + \sum_{i<j<k} C_{ijk} x_i x_j x_k + \dots + \sum_{i<j<k<l<\dots} C_{ijkl\dots} x_i x_j x_k x_l \dots,$$

where $x_i \in \{0, 1\}$ are binary variables. C_i, C_{ij}, C_{ijk}, $C_{ijkl}, C_{ijkl\dots}$are the corresponding coefficients. For example, consider the following HUBO objective function:

$$E = 2x_1 - 3x_2 + x_4 - 4x_1x_2 + x_1x_3 + 2x_1x_2x_3 - x_2x_3x_4 + 5.$$

Here, $x_1, x_2, x_3, x_4 \in \{0, 1\}$ are binary variables, and the coefficients correspond to each term in the expression. The HUBO problem is to find $x_1, x_2, x_3, x_4 \in \{0, 1\}$ that minimize $E(x_1, x_2, x_3, x_4)$. Exhaustive evaluation of all 2^4 assignments yields the minimum value of E is 0, attained uniquely at $(x_1, x_2, x_3, x_4) = (1, 1, 0, 0)$.

B. Graph Embedding Problem

Graph embedding has a wide range of applications in various fields, such as supercomputer [3], [4], network analysis [5], [6], pattern recognition [7], [8], and quantum computing [9]. There have been many studies on graph embedding [11], [12]. We define a graph $G(V, E)$ as a mathematical data structure consisting of a node set $V = \{v_1, v_2, v_3, \dots, v_n\}$ and an edge set E. Each edge $e_{ij} \in E$ represents a connection between two distinct nodes v_i and v_j. It can be denoted as (v_i, v_j) where $v_i, v_j \in V$. In this case, v_i and v_j are referred to as adjacent nodes.

Given a guest graph $G = (V_G, E_G)$ and a host graph $H = (V_H, E_H)$, the problem of finding an embedding defined by a mapping $f : V_G \to V_H$ is called the graph embedding problem. As an illustrative example, we consider the 4-dimensional hypercube graph as the guest graph. The complete graph with 12 vertices serves as the host graph, as shown in Fig. 1. In this example, the node set of the guest graph is defined as $V_G = \{u_i \mid u_i \in V_G, i = 0, 1, \dots 7\}$. The vertex set of the host graph is defined as $V_H = \{v_j \mid u_j \in V_H, j = 0, 1, \dots 11\}$. The result of the vertex embedding is given by the mapping $f(u_0) = v_3, f(u_1) = v_7, f(u_2) = v_{11}, f(u_3) = v_6, f(u_4) = v_5, f(u_5) = v_0, f(u_6) = v_2, f(u_7) = v_1$. Correspondingly, the edge embedding is given by the mapping:

$$
\begin{aligned}
&(u_0, u_1) \to (v_3, v_7), \quad &(u_0, u_3) \to (v_3, v_6),\\
&(u_0, u_4) \to (v_3, v_5), \quad &(u_1, u_2) \to (v_7, v_{11}),\\
&(u_1, u_7) \to (v_1, v_7), \quad &(u_2, u_3) \to (v_6, v_{11}),\\
&(u_2, u_6) \to (v_2, v_{11}), \quad &(u_3, u_5) \to (v_0, v_6),\\
&(u_4, u_5) \to (v_0, v_5), \quad &(u_4, u_7) \to (v_1, v_5),\\
&(u_5, u_6) \to (v_0, v_2), \quad &(u_6, u_7) \to (v_1, v_2).
\end{aligned}
$$

Since $E_{f(v_G)} \subseteq E_H$, the embedding is successful.

C. Graph Embedding Problem With Optical Switch

If we extract certain edges from the host graph as shown in Fig. 2, the embedding of the guest graph fails for the edges (u_0, u_3) and (u_4, u_7), since $(f(u_0), f(u_3)), (f(u_4), f(u_7)) \notin E_H$. In this situation, we attach an optical switch to host graph to make the optical switch connect to every node of host graph so that one edge among those connected to each node of guest graph can be embedded for optical switch. In Fig. 2, even if $(f(u_0), f(u_3)), (f(u_4), f(u_7)) \notin E_H$, (u_0, u_3) and (u_4, u_7) can be embedded into host graph.

II. QUBO MODEL AND HUBO MODEL OF GRAPH EMBEDDING PROBLEM

For the guest graph $G = (V_G, E_G)$ and the host graph $H = (V_H, E_H)$, we define the binary variables as follows.

$$
x_{u,v} = \begin{cases} 1 & \text{if } u \in V_G \text{ is mapped to } v \in V_H, \\ 0 & \text{otherwise.} \end{cases} \tag{1}
$$

The values of the binary variables corresponding to the graph embedding in Fig. 1 are summarized in Tab. I.

TABLE I: Binary variables of the graph embedding in Fig. 1.

$x_{u,v}$	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
v_0	0	0	0	0	0	1	0	0
v_1	0	0	0	0	0	0	0	1
v_2	0	0	0	0	0	0	1	0
v_3	1	0	0	0	0	0	0	0
v_4	0	0	0	0	0	0	0	0
v_5	0	0	0	0	1	0	0	0
v_6	0	0	0	1	0	0	0	0
v_7	0	1	0	0	0	0	0	0
v_8	0	0	0	0	0	0	0	0
v_9	0	0	0	0	0	0	0	0
v_{10}	0	0	0	0	0	0	0	0
v_{11}	0	0	1	0	0	0	0	0

As shown in Tab. I, each column contains a single entry equal to 1. Such a constraint, in which a vector contains exactly one entry equal to 1 while all other entries are 0, is called a one-hot constraint. In Tab. I, each row contains at most a single entry equal to 1. Such a constraint, in which the elements of a vector are binary with at most one element equal to 1, is called a zero-one-hot constraint.

If $(u, u') \in E_G$ is mapped into $(v, v') \in E_H$, as shown in Fig. 3, there are two alternative node embeddings can be represented as follows:

$$
\begin{aligned}
&\text{(i)} u \mapsto v, u' \mapsto v', x_{u,v} x_{u',v'} = 1;\\
&\text{(ii)} u \mapsto v', u' \mapsto v, x_{u,v'} x_{u',v} = 1.
\end{aligned}
$$

Therefore, if $x_{u,v} x_{u',v'} + x_{u,v'} x_{u',v} = 1$, $(u, u') \in E_G$ is mapped to $(v, v') \in E_H$; otherwise, $x_{u,v} x_{u',v'} + x_{u,v'} x_{u',v} = 0$.

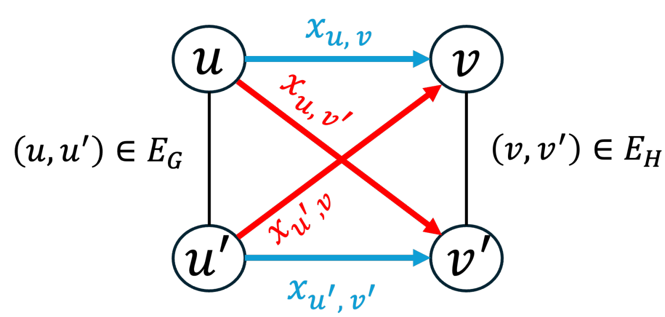

Fig. 3: Two alternative node embeddings for the mapping $(u, u') \mapsto (v, v')$.

A. Constraints For Graph Embedding Problem

Every vertex in the guest graph is mapped to a unique vertex in the host graph, therefore

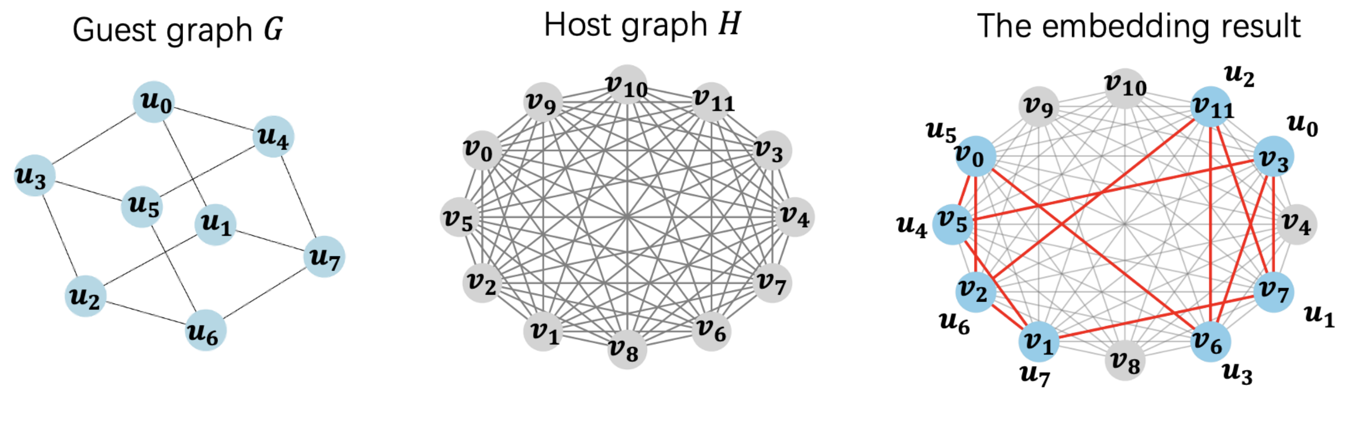

Fig. 1: Example of graph embedding without an optical switch.

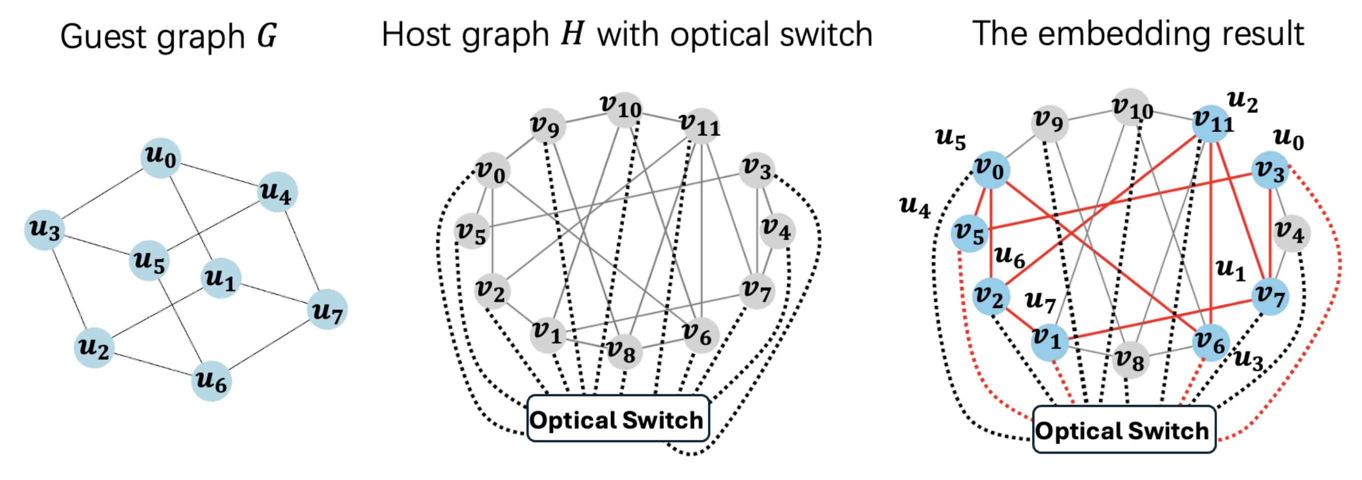

Fig. 2: Example of graph embedding with an optical switch.

$$\sum_{v \in V_H} x_{u,v} = 1. \tag{2}$$

It satisfies the one-hot constraint, therefore we obtain

$$C_1 = \sum_{u \in V_G} (1 - \sum_{v \in V_H} x_{u,v})^2. \tag{3}$$

If $C_1 = 0$, each node of the guest graph is mapped to only one node in the host graph; otherwise, at least one node of the guest graph is mapped to either zero or more than two nodes in the host graph. Since for every $v \in V_H$, there is at most one node $u \in V_G$ mapped to it, i.e., either no node or exactly one node u satisfies $f(u) = v$,

$$\sum_{u \in V_G} x_{u,v} = 0, 1. \tag{4}$$

It satisfies the zero-one-hot constraint, therefore we obtain

$$C_2 = \sum_{v \in V_H} (1 - \sum_{u \in V_G} x_{u,v}) \sum_{u \in V_G} x_{u,v}. \tag{5}$$

If $C_2 = 0$, each node of the host graph is mapped to zero or one node in the guest graph; otherwise, at least one node of the host graph is mapped to more than two nodes in the guest graph.

B. QUBO Model Of Graph Embedding without Optical Switch

An embedding of a guest graph $G = (V_G, E_G)$ into a host graph $H = (V_H, E_H)$ is regarded as successful if all edges of the guest graph are successfully embedded. It can be formulated as follows:

$$N_{E_G} = \sum_{(u,u') \in E_G} \sum_{(v,v') \in E_H} (x_{u,v} x_{u',v'} + x_{u,v'} x_{u',v}), \tag{6}$$

where N_{E_G} denotes the number of edges in the guest graph. Then, we suppose

$$C_3 = N_{E_G} - \sum_{(u,u') \in E_G} \sum_{(v,v') \in E_H} (x_{u,v} x_{u',v'} + x_{u,v'} x_{u',v}). \tag{7}$$

To minimize the objective function while satisfying the given constraints, we formulate the problem as follows:

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$$E = P_1(C_1 + C_2) + C_3, \tag{8}$$

where P_1 is penalty term. The optimal solution is obtained when $E = 0$. If all edges of graph G are successfully embedded into graph H, the $E = 0$; otherwise, E is equal to the number of edges that fail to be embedded.

C. QUBO Model Of Graph Embedding with Optical Switch

If attaching an optical switch to host graph ensures that, even if the embedding of one edge among those connected to each node of guest graph fails, a successful embedding can still be achieved for optical switch. Hence, an embedding of a guest graph $G = (V_G, E_G)$ into a host graph $H = (V_H, E_H)$ is regarded as successful if the number of edges among those connected to each node of guest graph successfully that are embedded is N_u or $N_u - 1$. Here, N_u represents the number of edges among those connected to $u \in V_G$, as shown in Fig. 4.

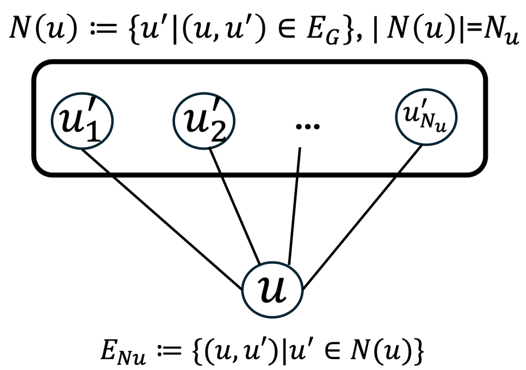

Fig. 4: The image of edges among those connected to $u \in V_G$.

It can be formulated as follows:

$$E_u = \sum_{u' \in N(u)} \sum_{(v,v') \in E_H} (x_{u,v}x_{u',v'} + x_{u,v'}x_{u',v}) = N_u, N_u - 1, \tag{9}$$

where N_u denotes the set of nodes adjacent to $u \in V_G$. To minimize the objective function while satisfying the given constraints, we consider all nodes of guest graph have to satisfy eq. 9, we obtain

$$C_4 = \sum_{u \in V_G} (E_u - N_u)(E_u - N_u + 1). \tag{10}$$

In eq. 10, There are cubic and quartic terms. Hence, we formulated HUBO model as follows:

$$E = P_2(C_1 + C_2) + C_4, \tag{11}$$

where P_2 is penalty term. The optimal solution is obtained when $E = 0$. If all edges of graph G are successfully embedded into graph H with optical switch, $E = 0$.

When $x_{u,v}x_{u',v'} + x_{u,v'}x_{u',v} = 1$, $(u, u') \in E_G$ is mapped to $(v, v') \in E_H$ without optical switch. The value of $\sum_{(u,u') \in E_G} \sum_{(v,v') \in E_H} (x_{u,v}x_{u',v'} + x_{u,v'}x_{u',v})$ represents the number of edges in the guest graph that are successfully embedded into the host graph without optical switch. Therefore, the value of the number of edges in the guest graph that are embedded into the host graph with optical switch is obtained by subtracting $\sum_{(u,u') \in E_G} \sum_{(v,v') \in E_H} (x_{u,v}x_{u',v'} + x_{u,v'}x_{u',v})$ from the number of edges in the guest graph. It can be formulated as follows:

$$N_{OSW} = N_{E_G} - \sum_{(u,u') \in E_G} \sum_{(v,v') \in E_H} (x_{u,v}x_{u',v'} + x_{u,v'}x_{u',v}),$$

where N_{OSW} is the number of edges in the guest graph that are embedded into the host graph with optical switch.

In Fig. 2, (u_0, u_3) and (u_4, u_7) use optical switch to be embedded into host graph. Other edges in guest graph can be embedding into host graph without optical switch. In fact, under the condition that at most one edge uses optical switch for each node in the guest graph, C_4 can still be 0, even if some edges other than (u_0, u_3) and (u_4, u_7) use optical switch. Therefore, for HUBO model of graph embedding with optical switch, there are cases where optical switch is used unnecessarily. When the value of N_{OSW} is minimized, the edges of guest graph use optical switch as little as possible. Therefore, We consider a HUBO model that minimizes the optical connections as follows:

$$E = P_3(C_1 + C_2 + C_4) + N_{OSW}, \tag{12}$$

where P_3 is penalty term. When $P_3(C_1 + C_2 + C_4)$ is zero, that is $E = N_{OSW}$, it indicates that the graph embedding with optical switch success. The value of E is the number of optical connections. If the value of E exceeds N_{OSW}, it indicates that the graph embedding with optical switch fails.

III. Experiment Results

We solve QUBO and HUBO instances defined by our formulations using Easy solver which can solve QUBO and HUBO problems [13], [14]. We run experiments on an Intel(R) Xeon(R) Platinum 8358 CPU@2.60GHz. All experiments' time limit was set to 500 seconds. Each graph embedding is executed 10 times. The results are averaged. For the guest and host graphs selected in this experiment, the penalty terms P_1, P_2 and P_3 were set to 1000. The experimental tables list the numbers of variables, linear terms, quadratic terms, cubic terms, and quartic terms, which reflect the size and complexity of the graph embedding problem. As the size of the problem increases, the difficulty of solving it also increases.

We have three scenarios, A, B, and C as follows:

- Scenario A: the QUBO model for graph embedding without optical switch, in which E is defined in Eq.8
- Scenario B: the HUBO model for graph embedding with optical switch, in which E is defined in Eq.11
- Scenario C: the HUBO model for graph embedding that minimizes optical connections, in which E is defined in Eq.12

We describe the guest and host graphs selected for our experiments. The random regular graphs is chosen as the host graph, in which every node has the same degree. A δ-regular graph means that each vertex of the graph has δ edges. The degree of the host graph is fixed, while the number of vertices is increased in steps of 10. Hypercube graph and Torus graph are chosen as the guest graph. A hypercube graph is a graph whose vertices represent all binary strings of a given length, with edges connecting vertices that differ in exactly one bit. In our experiments, we use 4-dimensional and 5-dimensional hypercube graphs. The 4-dimensional hypercube graph consists of 16 nodes and 32 edges, where each node is connected to 4 others. Similarly, the 5-dimensional hypercube graph has 32 nodes and 80 edges, with each node connected to 5 others. A torus graph is a grid-like graph with periodic boundary connections, extending to two or more dimensions. We use 2-dimensional torus graphs of size 6×3 and 3-dimensional torus graphs of size $4 \times 4 \times 2$. The 2-dimensional torus graph of size 6×3 consists of 18 nodes and 36 edges, where each node is connected to four neighbors in a wrapped 6×3 grid. The 3-dimensional torus graph of size $4 \times 4 \times 2$ consists of 32 nodes and 96 edges, where each vertex is connected to six neighbors, with edges wrapping around in all three dimensions. All the host graphs we selected have larger degrees and more nodes than the guest graphs. Therefore, they are theoretically all embeddings can be successfully in theory.

The experimental results for the 4-dimensional hypercube graph are shown in Tables II to V. When the degree of the host graph is 9, the optimum can be found in Scenarios A, B and C for all host graphs with the number of nodes ranging from 40 to 100. When the degree of the host graph is 7 and 8, the optimum can not be found in Scenario A. However, the optimum can be found in Scenarios B and C. It indicates that the HUBO models attaching optical switch to host graph appears to be more effective than the QUBO model without optical switch. When the degree of the host graph is 6, the optimum can not be found in Scenarios A, B and C.

The experimental results for the 5-dimensional hypercube graph are shown in Tables VI to VIII. When the degree of the host graph is 15, the optimum can be found in Scenarios A for all host graphs with the number of nodes ranging from 40 to 100. The optimum was found in Scenario B that the number of nodes of host graph is 90. However, the optimum can not be found in Scenario C. When the degree of the host graph is 14, the optimum can not be found in Scenarios A and C. However, the optimum can be found only in Scenario B that the number of nodes of host graph is 40 and 60. When the degree of the host graph is 13, the optimum can not be found in Scenarios A, B and C. If the graph is large, it's difficult to find the optimum for all Scenarios.

The experimental results for the 6×3 torus graph are shown in Tables IX to XII. When the degree of the host graph is 7, the optimum can be found in Scenarios A, B and C for all host graphs with the number of nodes ranging from 40 to 100. When the degree of the host graph is 6 and 5, the optimum can not be found in Scenario A. However, the optimum can be found in Scenarios B and C, except for the number of nodes of host graph is 40 in Scenario C. When the degree of the host graph is 4, the optimum can not be found in Scenarios A, B and C.

The experimental results for the $4 \times 4 \times 2$ torus graph are shown in Tables XIII to XV. When the degree of the host graph is 17, the optimum can be found in Scenario A for all host graphs with the number of nodes ranging from 40 to 100. The optimum was found in Scenario B that the number of nodes of host graph is 40, 70 and 80. However, the optimum can not be found in Scenarios A and C. When the degree of the host graph is 16 and 15, the optimum can not be found in Scenarios A, B and C.

From all the experimental results, we observe that the HUBO models attaching optical switch to host graph appears to be more effective than the QUBO model without optical switch. As the size of the problem increases, the difficulty of solving it also increases. The performance of the HUBO model that minimizes optical connection is worse that that of the HUBO model that does not minimize optical connection.

IV. Conclusion

We have proposed QUBO formulation for solving the graph embedding problem without optical switch, HUBO formulation for solving the graph embedding problem with optical switch and the formulation to minimize the optical connection. We have shown the experiment results using Easy solver. We can solve the graph embedding problem. Experiment results show that the model with optical switch can solve more graph embedding problem that the model without optical switch. We can minimize optical connection as much as possible by the HUBO model we proposed. However, the performance of it is worse that that of the HUBO model that does not minimize optical connection. As the size of the problem increases, the difficulty of solving it also increases.

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TABLE II: Guest graph as a 4-dimensional hypercube graph and host graph as a random 6-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	7	4	155.8	7	4	146.2	7	4	166	7	4	146.4
Optical connection	——	8.5	5.8	——	8	6.2	——	9	6	——	8	6.4
TTS (s)	1.179	5.442	90.321	0.891	16.265	194.454	1.361	4.478	133.952	1.198	6.324	155.080
Variables	640	640	640	800	800	800	960	960	960	1120	1120	1120
Linear terms	640	640	640	800	800	800	960	960	960	1120	1120	1120
Quadratic terms	24960	24960	24960	35200	35200	35200	47040	47040	47040	60480	60480	60480
Cubic terms	——	176640	176640	——	220800	220800	——	264960	264960	——	309120	309120
Quartic terms	——	6087680	6087680	——	9625600	9625600	——	13969920	13969920	——	19120640	19120640

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	7	4	172.5	7	4	172.7	7	4	189.2
Optical connection	——	8.5	5.8	——	9	6	——	8.5	5.8
TTS (s)	0.948	9.117	95.716	2.221	7.263	53.179	4.028	9.314	207.396
Variables	1280	1280	1280	1440	1440	1440	1600	1600	1600
Linear terms	1280	1280	1280	1440	1440	1440	1600	1600	1600
Quadratic terms	75520	75520	75520	92160	92160	92160	110400	110400	110400
Cubic terms	——	353280	353280	——	397440	397440	——	441600	441600
Quartic terms	——	25077760	25077760	——	31841280	31841280	——	39411200	39411200

TABLE III: Guest graph as a 4-dimensional hypercube graph and host graph as a random 7-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	2	0	4.1	2	0	3.7	2	0	4.3	2	0	4.2
Optical connection	——	7.4	4.1	——	7.4	3.7	——	7.2	4.3	——	7.6	4.2
TTS (s)	0.067	5.442	15.066	0.030	16.265	36.793	0.134	4.478	26.730	0.054	6.324	27.437
Variables	640	640	640	800	800	800	960	960	960	1120	1120	1120
Linear terms	640	640	640	800	800	800	960	960	960	1120	1120	1120
Quadratic terms	26240	26240	26240	36800	36800	36800	48960	48960	48960	62720	62720	62720
Cubic terms	——	241920	241920	——	302400	302400	——	362880	362880	——	423360	423360
Quartic terms	——	8301440	8301440	——	13120800	13120800	——	19037760	19037760	——	26052320	26052320

# of nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	2	0	4.7	2	0	4	2	0	4.3
Optical connection	——	7.6	4.7	——	7.4	4	——	7.2	4.3
TTS (s)	0.116	9.117	26.935	0.158	7.263	25.336	0.136	9.314	29.259
Variables	1280	1280	1280	1440	1440	1440	1600	1600	1600
Linear terms	1280	1280	1280	1440	1440	1440	1600	1600	1600
Quadratic terms	78080	78080	78080	95040	95040	95040	113600	113600	113600
Cubic terms	——	483840	483840	——	544320	544320	——	604800	604800
Quartic terms	——	34164480	34164480	——	43374240	43374240	——	53681600	53681600

TABLE IV: Guest graph as a 4-dimensional hypercube graph and host graph as a random 8-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	4	0	4.9	4	0	6.2	4	0	5.6	4	0	6.2
Optical connection	——	7.8	4.9	——	7.7	6.2	——	7.9	5.6	——	7.7	6.2
TTS (s)	1.205	5.442	44.154	0.412	16.265	49.042	4.831	4.478	43.191	6.448	6.324	35.939
Variables	640	640	640	800	800	800	960	960	960	1120	1120	1120
Linear terms	640	640	640	800	800	800	960	960	960	1120	1120	1120
Quadratic terms	27520	27520	27520	38400	38400	38400	50880	50880	50880	64940	64940	64940
Cubic terms	——	317440	317440	——	396800	396800	——	476160	476160	——	555520	555520
Quartic terms	——	10644480	10644480	——	16889600	16889600	——	24568320	24568320	——	33680640	33680640

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	4	0	6.7	4	0	6.7	4	0	7.5
Optical connection	——	7.9	6.7	——	7.6	6.7	——	8	7.5
TTS (s)	5.282	9.117	28.659	2.577	7.263	29.508	2.969	9.314	15.982
Variables	1280	1280	1280	1440	1440	1440	1600	1600	1600
Linear terms	1280	1280	1280	1440	1440	1440	1600	1600	1600
Quadratic terms	80640	80640	80640	97920	97920	97920	116800	116800	116800
Cubic terms	——	634880	634880	——	714240	714240	——	793600	793600
Quartic terms	——	44226560	44226560	——	56206080	56206080	——	69619200	69619200

TABLE V: Guest graph as a 4-dimensional hypercube graph and host graph as a random 9-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	0	0	1.8	0	0	3.2	0	0	3.1	0	0	4
Optical connection	——	6.6	1.8	——	6.7	3.2	——	7.1	3.1	——	7.7	4
TTS (s)	0.037	2.233	29.689	0.024	5.235	29.987	0.077	5.131	38.506	0.151	7.759	25.672
Variables	640	640	640	800	800	800	960	960	960	1120	1120	1120
Linear terms	640	640	640	800	800	800	960	960	960	1120	1120	1120
Quadratic terms	28800	28800	28800	40000	40000	40000	52800	52800	52800	67200	67200	67200
Cubic terms	——	403200	403200	——	504000	504000	——	604800	604800	——	705600	705600
Quartic terms	——	13537920	13537920	——	21458400	21458400	——	31193280	31193280	——	42742560	42742560

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	0	0	4.2	0	0	3.7	0	0	2.4
Optical connection	——	7.9	4.2	——	7.2	3.7	——	7.1	2.4
TTS (s)	0.084	7.378	11.570	0.244	9.146	29.880	0.167	17.234	34.565
Variables	1280	1280	1280	1440	1440	1440	1600	1600	1600
Linear terms	1280	1280	1280	1440	1440	1440	1600	1600	1600
Quadratic terms	83200	83200	83200	100800	100800	100800	120000	120000	120000
Cubic terms	——	806400	806400	——	907200	907200	——	1008000	1008000
Quartic terms	——	56106240	56106240	——	71284320	71284320	——	88276800	88276800

TABLE VI: Guest graph as a 5-dimensional hypercube graph and host graph as a random 13-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	4	0.8	308.8	4	0.4	288.9	4	0.8	299.5	3.8	2.2	282.1
Optical connection	——	13.1	8	——	12.2	8.9	——	14.1	9.5	——	14.7	12.1
TTS (s)	8.643	190.945	52.462	11.065	149.153	50.283	1.014	53.998	38.506	18.849	124.307	69.188
Variables	1280	1280	1280	1600	1600	1600	1920	1920	1920	1120	1120	1120
Linear terms	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Quadratic terms	86400	86400	86400	116000	116000	116000	148800	148800	148800	185760	185760	185760
Cubic terms	——	2662400	2662400	——	3328000	3328000	——	3993600	3993600	——	4659200	4659200
Quartic terms	——	88017600	88017600	——	140442000	140442000	——	205034400	205034400	——	281794800	281794800

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	4	1.2	390.4	4	0.8	272.9	4	0.4	332.1
Optical connection	——	14.1	10.4	——	14.9	12.9	——	14.4	12.1
TTS (s)	5.939	177.239	63.528	5.657	261.782	50.795	1.657	200.858	57.784
Variables	2560	2560	2560	2880	2880	2880	3200	3200	3200
Linear terms	2560	2560	2560	2880	2880	2880	3200	3200	3200
Quadratic terms	224960	224960	224960	266400	266400	266400	312000	312000	312000
Cubic terms	——	5324800	5324800	——	5990400	5990400	——	6656000	6656000
Quartic terms	——	370723200	370723200	——	471819600	471819600	——	585084000	585084000

TABLE VII: Guest graph as a 5-dimensional hypercube graph and host graph as a random 14-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	9.7	0	14.3	11.4	2.4	391.7	5.7	0	108.9	11.5	6.4	431.3
Optical connection	——	14.8	13.9	——	12.6	11.7	——	11.5	10	——	16.1	11.3
TTS (s)	141.021	109.355	72.059	62.714	175.450	63.899	85.493	72.947	52.779	83.631	246.074	73.496
Variables	1280	1280	1280	1600	1600	1600	1920	1920	1920	1120	1120	1120
Linear terms	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Quadratic terms	89600	89600	89600	120000	120000	120000	166400	166400	166400	1913600	191360	191360
Cubic terms	——	3091200	3091200	——	3864000	3864000	——			——	5409600	5409600
Quartic terms	——	99344000	99344000	——	159460000	159460000	——			——	322028000	322028000

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	11.3	5.6	472	11.5	7.2	493.3	11.4	7.6	522.2
Optical connection	——	16.1	12	——	16.8	13.3	——	16.8	12.2
TTS (s)	82.590	277.691	80.031	92.667	227.184	65.484	87.121	266.865	70.839
Variables	2560	2560	2560	2880	2880	2880	3200	3200	3200
Linear terms	2560	2560	2560	2880	2880	2880	3200	3200	3200
Quadratic terms	230400	230400	230400	273600	273600	273600	320000	320000	320000
Cubic terms	——	6182400	6182400	——	6955200	6955200	——	7728000	6656000
Quartic terms	——	424480000	424480000	——	541044000	541044000	——	671720000	671720000

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TABLE VIII: Guest graph as a 5-dimensional hypercube graph and host graph as a random 15-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	0	0.2	167.5	0	1.2	158	0	0.2	338.6	0	0.2	329.7
Optical connection	——	10.9	7.5	——	11.9	8	——	14	8.6	——	13.8	9.7
TTS (s)	83.113	77.923	58.674	58.250	131.434	46.487	85.493	72.947	52.779	22.927	142.86	67.711
Variables	1280	1280	1280	1600	1600	1600	1920	1920	1920	1120	1120	1120
Linear terms	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Quadratic terms	92800	92800	92800	124000	124000	124000	158400	158400	158400	196000	196000	196000
Cubic terms	——	3552000	3552000	——	4440000	4440000	——	5328000	5328000	——	6216000	6216000
Quartic terms	——	115032000	115032000	——	184506000	184506000	——	270007200	270007200	——	371708400	371708400

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	0	0.6	247.5	0	0	380.9	0	2	280.3
Optical connection	——	14.6	7.5	——	14.2	10.9	——	15.2	10.3
TTS (s)	77.391	221.277	54.019	92.667	227.184	65.484	53.538	208.480	55.936
Variables	2560	2560	2560	2880	2880	2880	3200	3200	3200
Linear terms	2560	2560	2560	2880	2880	2880	3200	3200	3200
Quadratic terms	236800	236800	236800	280800	280800	280800	328000	328000	328000
Cubic terms	——	7104000	7104000	——	7992000	7992000	——	8880000	8880000
Quartic terms	——	489609600	489609600	——	623710800	623710800	——	774012000	774012000

TABLE IX: Guest graph as a 6×3 torus graph and host graph as a random 4-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	8	4	187.4	8	4	108.2	8	4	128.1	8	4	158.2
Optical connection	——	9.8	7.4	——	9.3	8.2	——	8.9	8.1	——	9.5	8.2
TTS (s)	1.978	3.019	4.256	2.064	4.892	140.326	2.249	19.021	142.966	10.334	19.890	226.756
Variables	720	720	720	900	900	900	1080	1080	1080	1260	1260	1260
Linear terms	720	720	720	900	900	900	1080	1080	1080	1260	1260	1260
Quadratic terms	25920	25920	25920	36900	36900	36900	49680	49680	49680	64260	64260	64260
Cubic terms	——	83520	83520	——	104400	104400	——	125200	125200	——	146160	146160
Quartic terms	——	3090240	3090240	——	4870800	4870800	——	7054560	7054560	——	9641520	9641520

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	8	4	147.9	8	4	167.6	8	4	207.5
Optical connection	——	9.4	7.9	——	9.3	7.6	——	9.3	7.5
TTS (s)	4.568	31.626	127.969	8.690	51.015	209.982	5.191	69.287	148.631
Variables	1440	1440	1440	1620	1620	1620	1800	1800	1800
Linear terms	1440	1440	1440	1620	1620	1620	1800	1800	1800
Quadratic terms	80640	80640	80640	98820	98820	98820	118800	118800	118800
Cubic terms	——	167040	167040	——	187920	187920	——	2088000	2088000
Quartic terms	——	12631680	12631680	——	16025040	16025040	——	19821600	19821600

TABLE X: Guest graph as a 6×3 torus graph and host graph as a random 5-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	4	0	16.9	4	0	5.7	4	0	7.1	4	0	6.6
Optical connection	——	7.5	6.9	——	8.5	5.7	——	8.8	7.1	——	8.7	6.6
TTS (s)	0.525	0.987	3.997	2.633	3.573	133.858	4.094	4.365	93.019	2.441	6.859	104.077
Variables	720	720	720	900	900	900	1080	1080	1080	1260	1260	1260
Linear terms	720	720	720	900	900	900	1080	1080	1080	1260	1260	1260
Quadratic terms	27360	27360	27360	38700	38700	38700	51840	51840	51840	66780	66780	66780
Cubic terms	——	133920	133920	——	167400	167400	——	200800	200800	——	234360	234360
Quartic terms	——	4816080	4816080	——	7595100	7595100	——	11004120	11004120	——	15043140	15043140

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	4	0	6.8	4	0	7.1	4	0	7.1
Optical connection	——	8.6	6.8	——	8.4	7.1	——	8.9	7.1
TTS (s)	4.685	3.233	104.51	5.113	11.727	200.163	8.157	9.542	144.075
Variables	1440	1440	1440	1620	1620	1620	1800	1800	1800
Linear terms	1440	1440	1440	1620	1620	1620	1800	1800	1800
Quadratic terms	83520	83520	83520	102060	102060	102060	122400	122400	122400
Cubic terms	——	267840	267840	——	301320	301320	——	334800	334800
Quartic terms	——	19712160	19712160	——	25011180	25011180	——	30940200	30940200

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TABLE XI: Guest graph as a 6×3 torus graph and host graph as a random 6-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	3	0	16.7	3	0	6.6	3	0	5	3	0	5.6
Optical connection	——	7.5	6.7	——	8.3	6.6	——	8.5	5	——	8.5	5.6
TTS (s)	0.556	1.015	3.444	1.270	3.886	11.741	8.783	4.775	155.975	15.319	3.286	132.218
Variables	720	720	720	900	900	900	1080	1080	1080	1260	1260	1260
Linear terms	720	720	720	900	900	900	1080	1080	1080	1260	1260	1260
Quadratic terms	28800	28800	28800	40500	40500	40500	54000	54000	54000	69300	69300	69300
Cubic terms	——	190080	190080	——	237600	237600	——	285120	285120	——	332640	332640
Quartic terms	——	6848640	6848640	——	10828800	10828800	——	15716160	15716160	——	21510720	21510720

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	3	0	6.3	3	0	6.4	3.2	0	5.7
Optical connection	——	8.3	6.3	——	8.4	6.4	——	8.2	5.7
TTS (s)	5.712	5.359	113.164	16.090	9.916	190.393	8.812	11.382	179.149
Variables	1440	1440	1440	1620	1620	1620	1800	1800	1800
Linear terms	1440	1440	1440	1620	1620	1620	1800	1800	1800
Quadratic terms	86400	86400	86400	105300	105300	105300	126000	126000	126000
Cubic terms	——	380160	380160	——	427680	427680	——	475200	475200
Quartic terms	——	28212480	28212480	——	35821440	35821440	——	44337600	44337600

TABLE XII: Guest graph as a 6×3 torus graph and host graph as a random 7-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	0	0	3.3	0	0	3.9	0	0	3.2	0	0	3.1
Optical connection	——	7.9	3.3	——	8.5	3.9	——	8.1	3.2	——	8.1	3.1
TTS (s)	1.256	1.741	141.886	0.816	1.908	94.658	1.036	7.488	207.033	1.672	6.848	150.969
Variables	720	720	720	900	900	900	1080	1080	1080	1260	1260	1260
Linear terms	720	720	720	900	900	900	1080	1080	1080	1260	1260	1260
Quadratic terms	30240	30240	30240	42300	42300	42300	56160	56160	56160	71820	71820	71820
Cubic terms	——	263520	263520	——	329400	329400	——	395280	395280	——	461160	461160
Quartic terms	——	9339120	9339120	——	14760900	14760900	——	21417480	21417480	——	29308860	29308860

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	0	0	4.5	0	0	6.2	0	0	5.2
Optical connection	——	8.2	4.5	——	8.8	6.2	——	8.4	5.2
TTS (s)	2.957	6.342	242.909	3.508	8.974	141.249	1.807	8.701	163.300
Variables	1440	1440	1440	1620	1620	1620	1800	1800	1800
Linear terms	1440	1440	1440	1620	1620	1620	1800	1800	1800
Quadratic terms	89280	89280	89280	108540	108540	108540	129600	129600	129600
Cubic terms	——	527040	527040	——	592920	592920	——	658800	658800
Quartic terms	——	38435040	38435040	——	48796020	48796020	——	60391800	60391800

TABLE XIII: Guest graph as a $4 \times 4 \times 2$ torus graph and host graph as a random 15-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	0.6	0.4	106.9	0.8	2	167	.2	2	198.5	0.4	8	276.8
Optical connection	——	11.4	6.9	——	11.7	6.7	——	13.8	8.5	——	13.3	10.2
TTS (s)	114.074	198.047	240.597	27.099	164.532	304.994	57.713	150.624	216.982	86.392	223.652	289.126
Variables	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Linear terms	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Quadratic terms	92800	92800	92800	124000	124000	124000	158400	158400	158400	196000	196000	196000
Cubic terms	——	3552000	552000	——	4440000	4440000	——	5328000	5328000	——	6216000	6216000
Quartic terms	——	115032000	115032000	——	184506000	184506000	——	270007200	270007200	——	371708400	371708400

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	0.4	1.3	226.3	1	3	260.833	0.2	0.4	194.7
Optical connection	——	15	9.7	——	15	10.8	——	14.6	11.3
TTS (s)	51.156	209.855	296.8	135.548	323.568	164.942	142.146	155.266	155.588
Variables	2560	2560	2560	2880	2880	2880	3200	3200	3200
Linear terms	2560	2560	2560	2880	2880	2880	3200	3200	3200
Quadratic terms	236800	236800	236800	280800	280800	280800	328000	328000	328000
Cubic terms	——	7104000	7104000	——	7992000	7992000	——	8880000	8880000
Quartic terms	——	489609600	489609600	——	623710800	623710800	——	774012000	774012000

TABLE XIV: Guest graph as a $4 \times 4 \times 2$ torus graph and host graph as a random 16-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	7.9	0.2	89.8	9.1	2.2	241.4	9.1	2.4	211.8	9.7	3	282.2
Optical connection	——	13.2	9.8	——	15.3	11.4	——	14.8	11.8	——	15.4	12.2
TTS (s)	13.991	109.820	89.800	158.723	208.183	212.580	202.867	298.510	216.967	55.081	274.823	278.167
Variables	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Linear terms	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Quadratic terms	96000	96000	96000	128000	128000	128000	163200	163200	163200	201600	201600	201600
Cubic terms	——	4044800	4044800	——	5056000	5056000	——	6067200	6067200	——	7078400	7078400
Quartic terms	——	127334400	127334400	——	205248000	205248000	——	301593600	301593600	——	416371200	416371200

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	9.3	4	281.5	9.4	3	280	9.8	15	313.2
Optical connection	——	15.2	14.8	——	15.2	13.3	——	4	13.2
TTS (s)	90.322	236.143	264.241	51.941	316.983	342.877	70.770	340.503	276.779
Variables	2560	2560	2560	2880	2880	2880	3200	3200	3200
Linear terms	2560	2560	2560	2880	2880	2880	3200	3200	3200
Quadratic terms	243200	243200	243200	288000	288000	288000	336000	336000	336000
Cubic terms	——	8089600	8089600	——	9100800	9100800	——	10112000	10112000
Quartic terms	——	549580800	549580800	——	70122240	70122240	——	871296000	871296000

TABLE XV: Guest graph as a $4 \times 4 \times 2$ torus graph and host graph as a random 17-regular graph

Nodes of host graph	40			50			60			70		
Scenario	A	B	C	A	B	C	A	B	C	A	B	C
E	0	0	25.1	0	0.4	106.2	0	1.2	158.3	0	0	190.1
Optical connection	——	10.4	5.1	——	10.4	5.2	——	11.9	8.3	——	12	10.1
TTS (s)	2.683	23.019	296.323	8.584	189.292	230.699	3.362	205.596	212.867	2.132	191.419	219.993
Variables	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Linear terms	1280	1280	1280	1600	1600	1600	1920	1920	1920	2240	2240	2240
Quadratic terms	99200	99200	99200	132000	132000	132000	168000	168000	168000	207200	207200	207200
Cubic terms	——	4569600	4569600	——	5712000	5712000	——	6854400	6854400	——	7996800	7996800
Quartic terms	——	144516800	144516800	——	233746000	233746000	——	342919200	342919200	——	472900400	472900400

Nodes of host graph	80			90			100		
Scenario	A	B	C	A	B	C	A	B	C
E	0	0	107.8	0	4.3	261.1	0	3	130
Optical connection	——	14.7	7.8	——	13.7	11.2	——	13.2	13.3
TTS (s)	2.392	293.329	280.670	8.004	198.177	336.288	7.376	228.180	274.020
Variables	2560	2560	2560	2880	2880	2880	3200	3200	3200
Linear terms	2560	2560	2560	2880	2880	2880	3200	3200	3200
Quadratic terms	249600	249600	249600	295200	295200	295200	344000	344000	344000
Cubic terms	——	9139200	9139200	——	10281600	10281600	——	11424000	11424000
Quartic terms	——	623689600	623689600	——	795286800	795286800	——	987692000	987692000

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